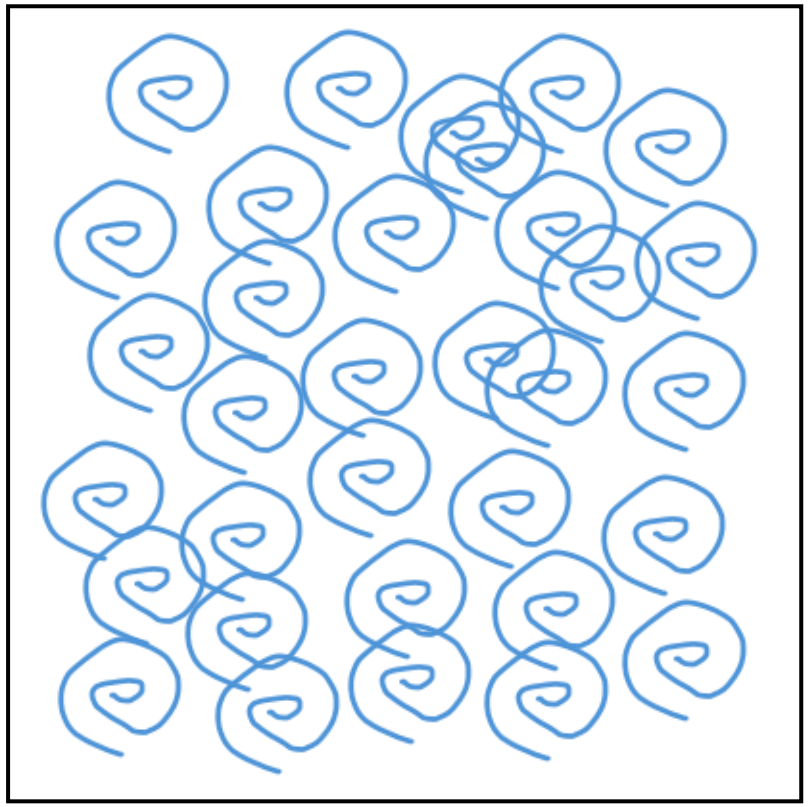
Scale dependence of Isopycnal Stirring Coefficients from Observational Estimates in the Kuroshio Extension

Tokyo University of Marine Science and Technology
Yuki Ikeda, Takeyoshi Nagai

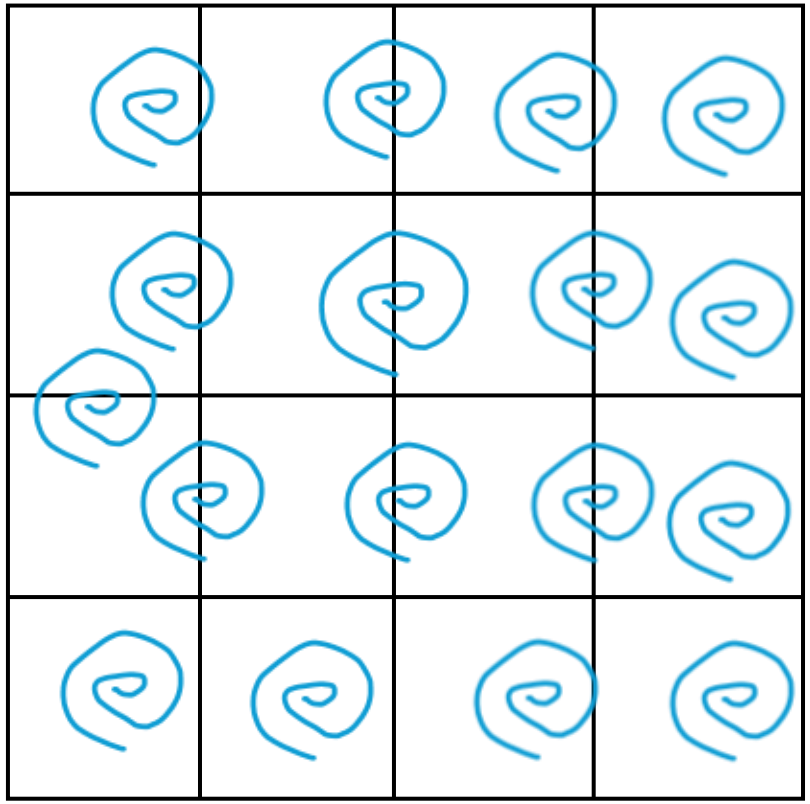
1. Research Background

Coarse resolution(1°) $\Delta X \gg L_{eddy}$



・There is a clear separation between eddies and mean flows.
・GM/Redi parameterization is used.

Gray zone (1/4°) $\Delta X \sim L_{eddy}$



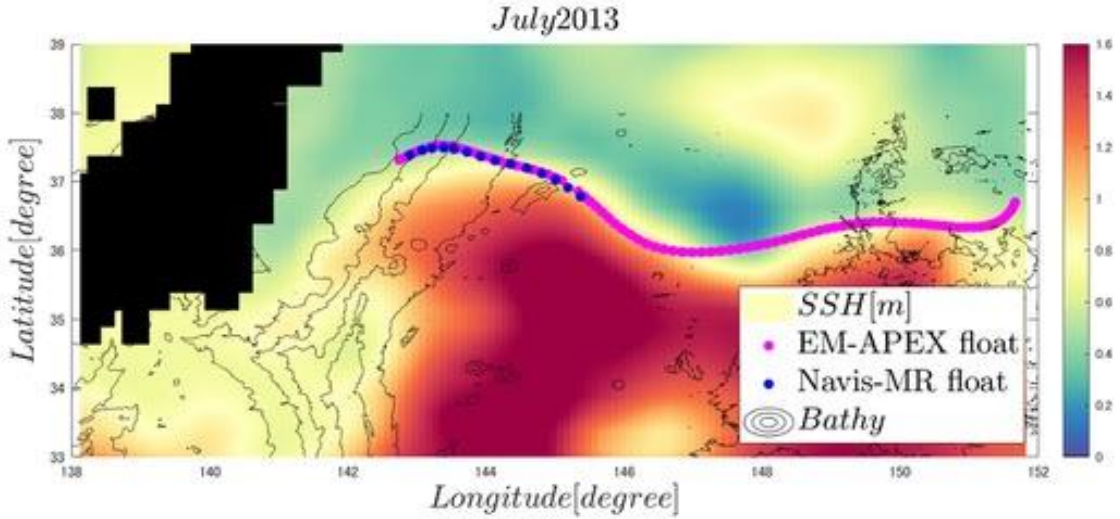
・The part of eddies are resolved.
・If GM/Redi parameterization is used, parameterization excessively dissipates the APE and eddy activity is suppressed.

2. The objective of this study

1. Quantifying the isopycnal stirring coefficient using fine-scale and microscale observational data and reanalysis data.

2. Clarifying the contribution of stirring by large-scale flow strain.

3. Study area and method



Navis Autonomous profiling float
Sampling frequency: 1Hz
Upward cast
Temp, Salt, Pressure

MicroRider
Sampling frequency : 512Hz
Shear probe
FP07 thermistor

EM-APEX float
Sampling frequency :
1/3-5s (temp,salt,pressure)
1/25.5s (velocity)
Both Upward and Downward cast

- **Mixing length theory(fine-scale observation)**

$$K_e^{fine} = c_e U_e L_{mix}$$

$$U_e = \sqrt{(u_{APEX} - \langle u_{APEX} \rangle)^2 + (v_{APEX} - \langle v_{APEX} \rangle)^2}$$

$$\theta_{rms} = \sqrt{\langle (\theta_{APEX} - \theta_m)^2 \rangle}$$

$$\begin{cases} c_e: \text{Mixing coefficient , we have used 0.16 value} \\ U_e: \text{Representative eddy velocity} \\ L_{mix} = \dfrac{\theta_{rms}}{\left|\nabla_{\parallel} \theta_m\right|}: \text{Mixing length} \end{cases}$$

- **Triple decomposition framework (microscale observation)**

$$K_e^{micro} = \frac{\frac{1}{2}\langle \chi \rangle - \langle K_h \rangle \left(\frac{\partial \theta_m}{\partial z_\perp}\right)^2}{\left|\nabla_{\parallel} \theta_m\right|^2}$$

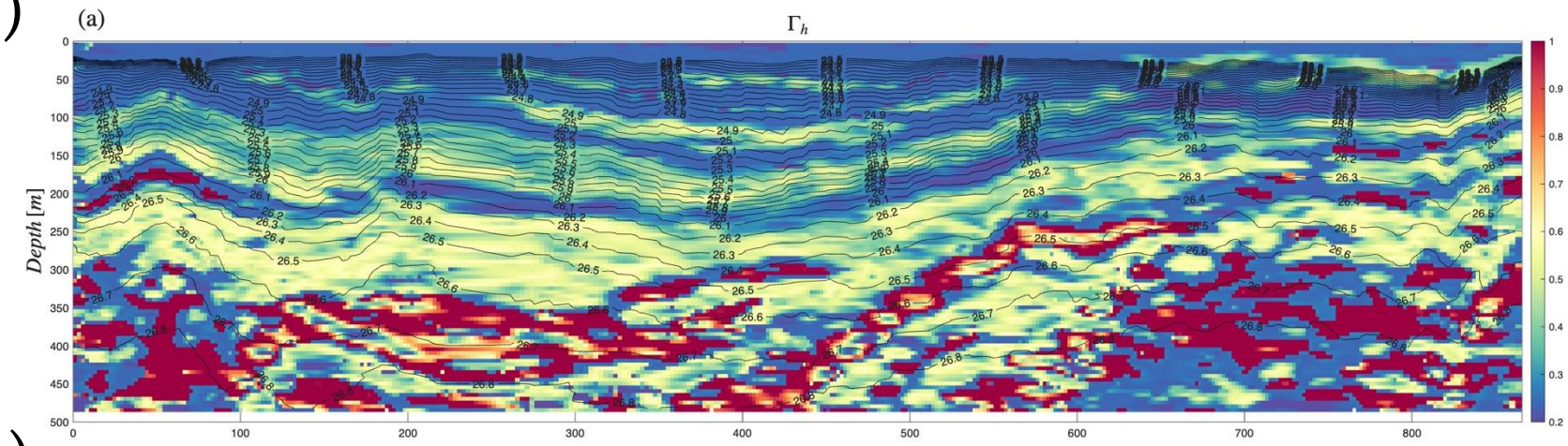
$$\begin{cases} \chi: Dissipation\ rate\ of\ microscale\ temp \\ \dfrac{\partial \theta_m}{\partial z_\perp}: Diapycnal\ gradient\ of\ resolved\ temp \\ \left|\nabla_{\parallel} \theta_m\right|: Isopycnal\ gradient\ of\ resolved\ temp \end{cases}$$

$K_h = \Gamma_h \frac{\varepsilon}{N^2}$ is the vertical heat diffusivity, and the heat mixing efficiency, Γ_h is derived using the turbulence model from Canuto et al.(2008,2010).

4. Results and Discussion

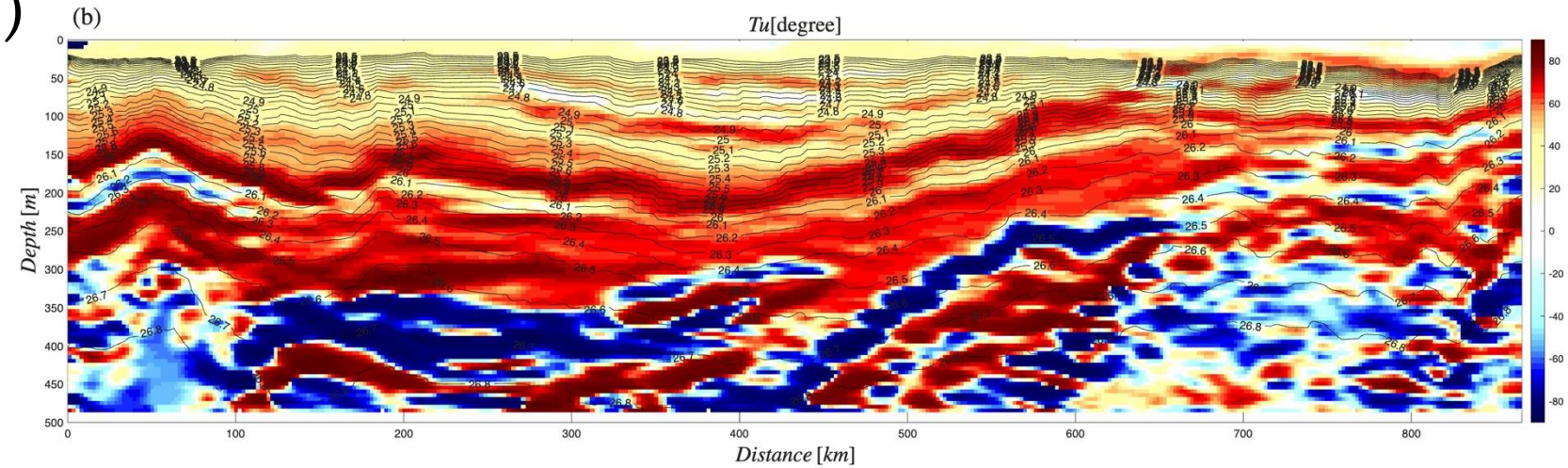
(a)

$\Gamma_h(R_\rho, R_i)$



(b)

Tu



$$\langle w_t b_t \rangle = g\alpha\langle w_t \theta_t \rangle \quad - g\beta\langle w_t s_t \rangle$$

Salt finger	+	−	<	+
Diffusive Convection	+	+	>	−

(a)

Diapycnal Heat diffusivity

$$K_h = \Gamma_h \frac{\varepsilon}{N^2}$$

(b)

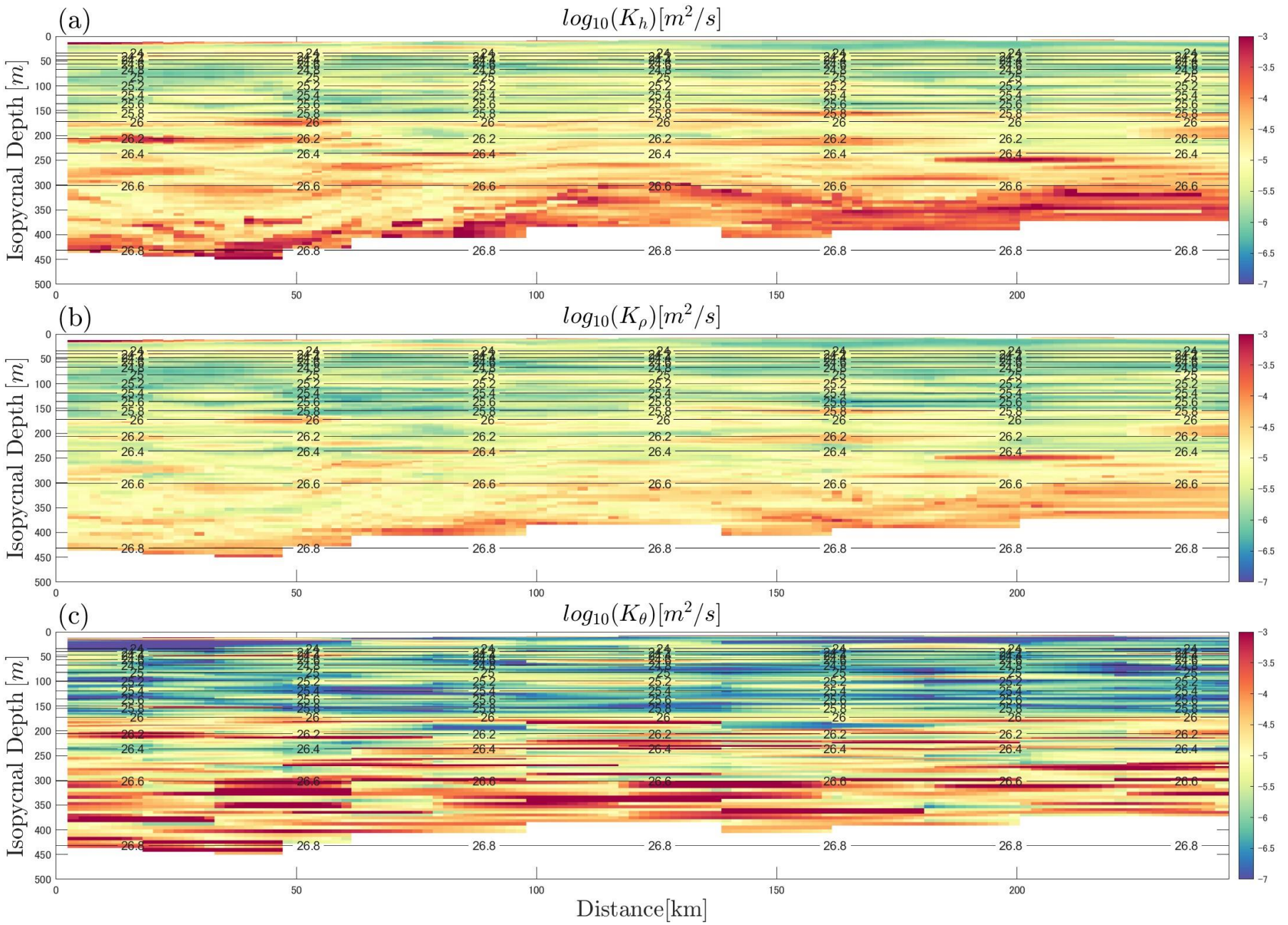
Diapycnal Density diffusivity

$$K_\rho = 0.2 \frac{\varepsilon}{N^2}$$

(c)

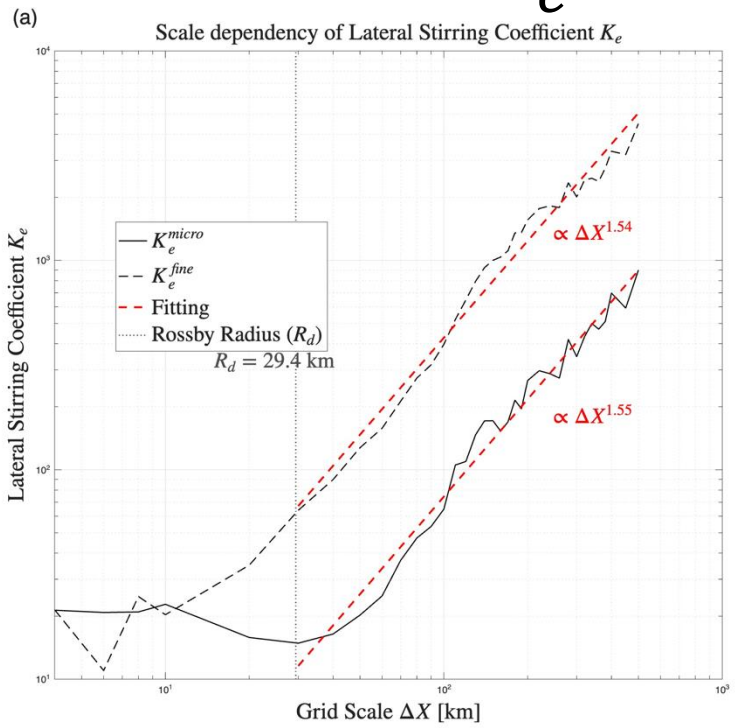
Diapycnal Heat diffusivity

$$K_\theta = \frac{\chi}{2\left(\frac{\partial \bar{\theta}}{\partial z}\right)^2}$$



Sub-grid scale Lateral and Isopycnal Stirring Coefficient

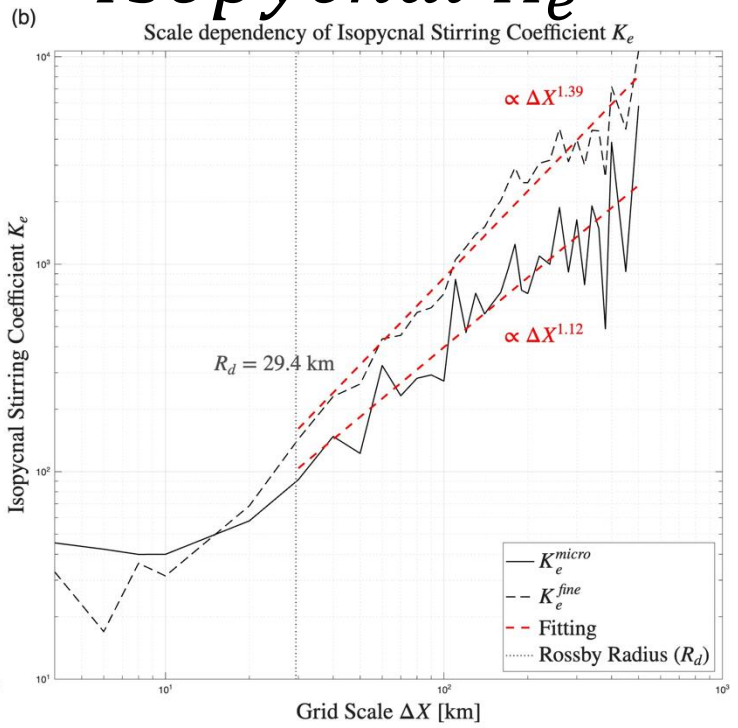
$Lateral\ K_e$



$$K_e^{fine} \propto \Delta X^{1.54}$$

$$K_e^{micro} \propto \Delta X^{1.55}$$

$Isopycnal\ K_e$



$$K_e^{fine} \propto \Delta X^{1.39}$$

$$K_e^{micro} \propto \Delta X^{1.12}$$

$$\left(\frac{\partial \theta}{\partial x}\right)_z = \left(\frac{\partial \theta}{\partial x}\right)_\sigma + \frac{\partial \theta}{\partial \sigma}\left(\frac{\partial \sigma}{\partial x}\right)_z$$

The discrepancy in magnitude between the fine-scale and microscale may be caused by the time lag in variance cascading from fine-scales to microscales.

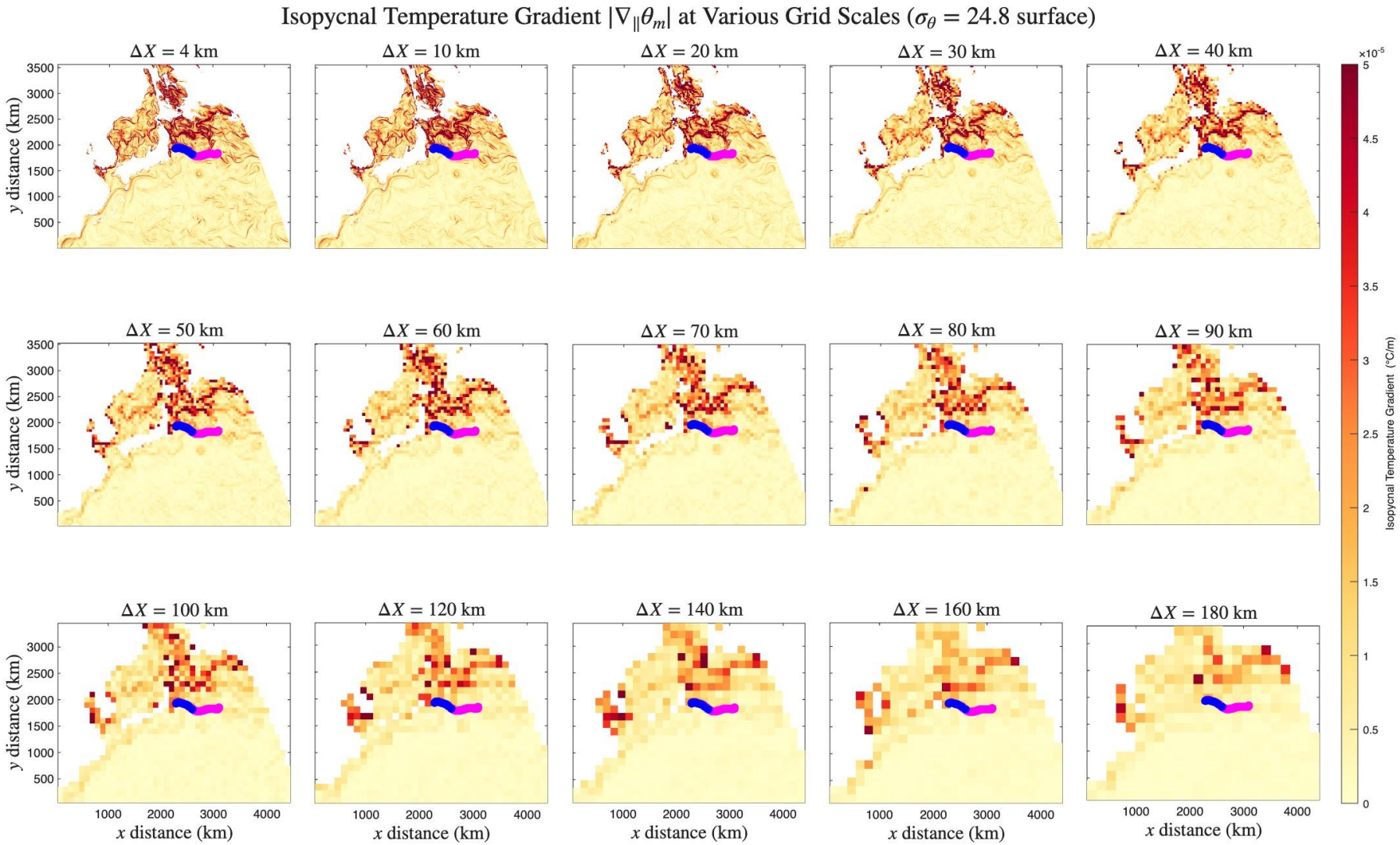
5. Summary

- The isopycnal stirring coefficient estimated from fine-scale observations showed a $\Delta X^{1.39}$scaling, while that from microscale observations showed a $\Delta X^{1.12}$ scaling. The values showed the same order of magnitude.

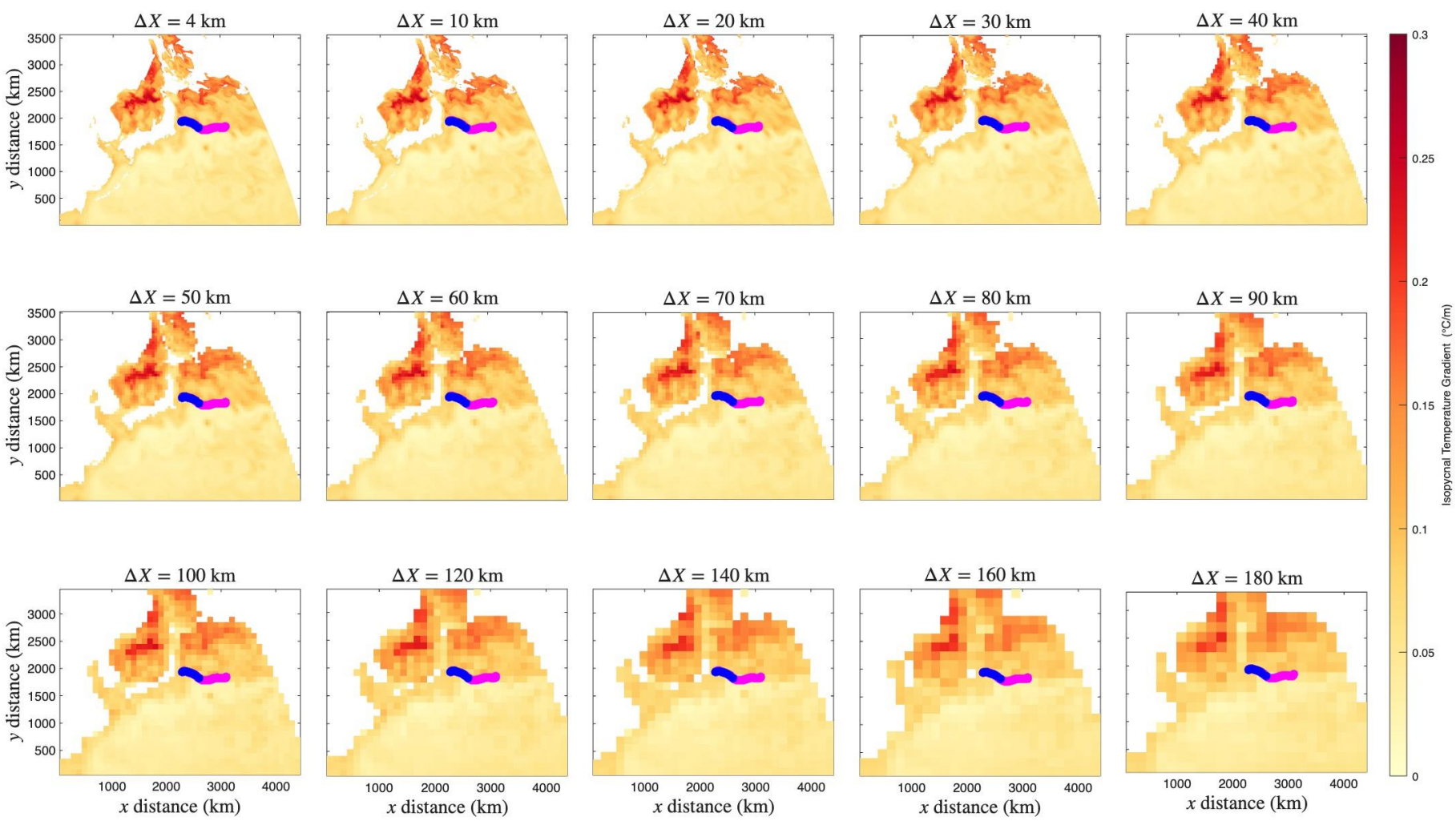
- Mixing length showed the $\Delta X^{0.69}$, which is flatter than ΔX^{1}.
  This may be caused by the leakage of resolved components into the sub-grid scale due to tracer curvature, suggesting the contribution of the non-local stirring term in the presence of background flow strain.

Supplementary

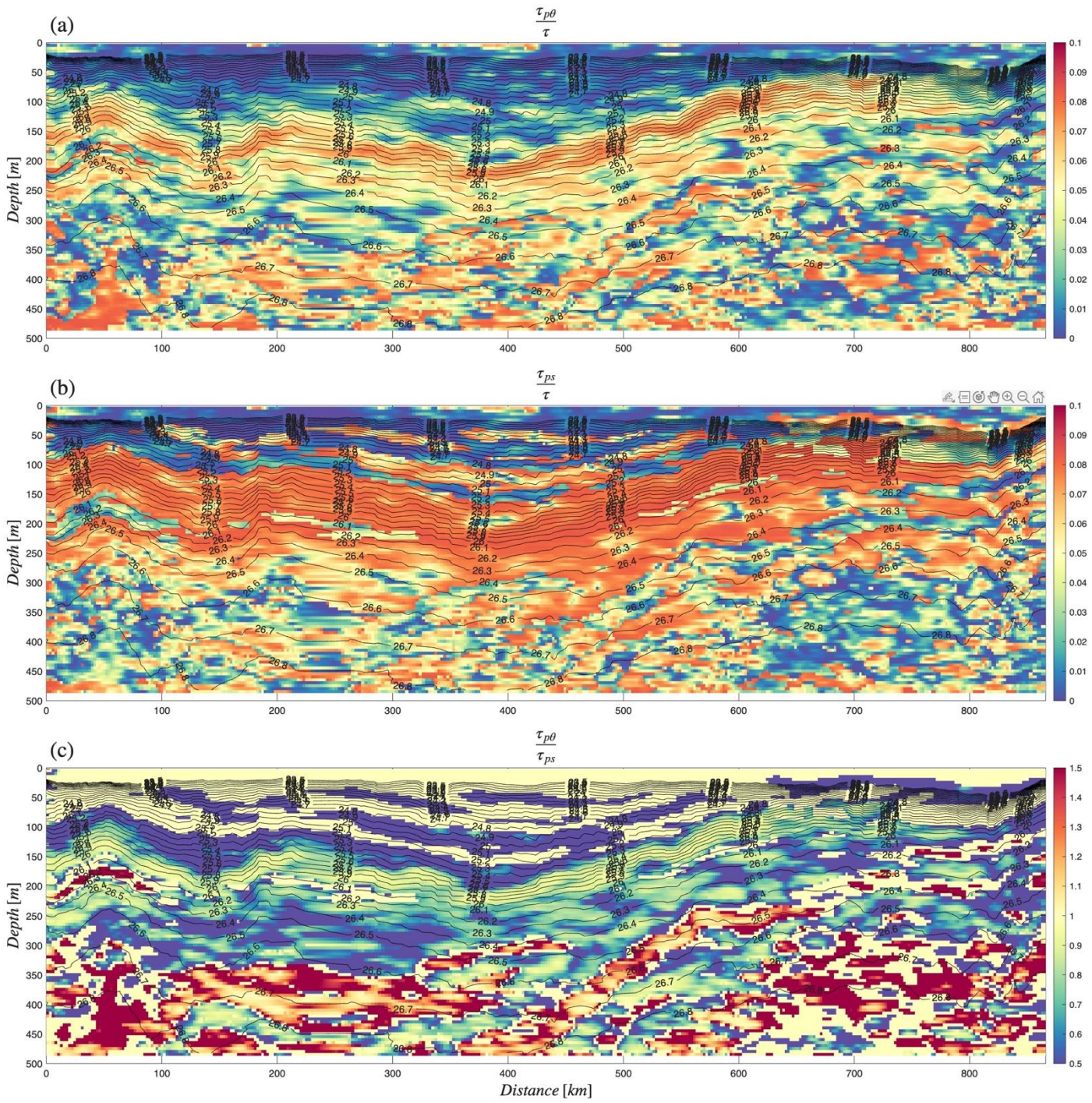
Isopycnal Temperature Gradient $|\nabla_{||}\theta_m|$ at various grid scales$(24.8\sigma_\theta)$



Diapycnal Temperature Gradient $\frac{\partial \theta_m}{\partial z_\perp}$ at various grid scales$(24.8\sigma_\theta)$



$$\frac{\partial \theta_m}{\partial z_\perp} = \frac{\partial \theta_m}{\partial z} - \frac{\partial z_\rho}{\partial x}\frac{\partial \theta_m}{\partial x} - \frac{\partial z_\rho}{\partial y}\frac{\partial \theta_m}{\partial y}$$



Vertical heat

$$\frac{DJ^h}{Dt} = -\overline{w^2}\frac{\partial T}{\partial z} + g\left(\alpha\overline{\theta^2} - \beta\overline{\theta s}\right) - \frac{J^h}{\tau_{p\theta}}$$

Vertical salinity

$$\frac{DJ^s}{Dt} = -\overline{w^2}\frac{\partial S}{\partial z} + g\left(\alpha\overline{\theta s} - \beta\overline{s^2}\right) - \frac{J^s}{\tau_{ps}}$$

Stable: $\tau_{ps} = \tau_{p\theta}$ dissipation

Salt finger : $\tau_{ps} > \tau_{p\theta}$ → $\frac{J^s}{\tau_{ps}}$ ↓

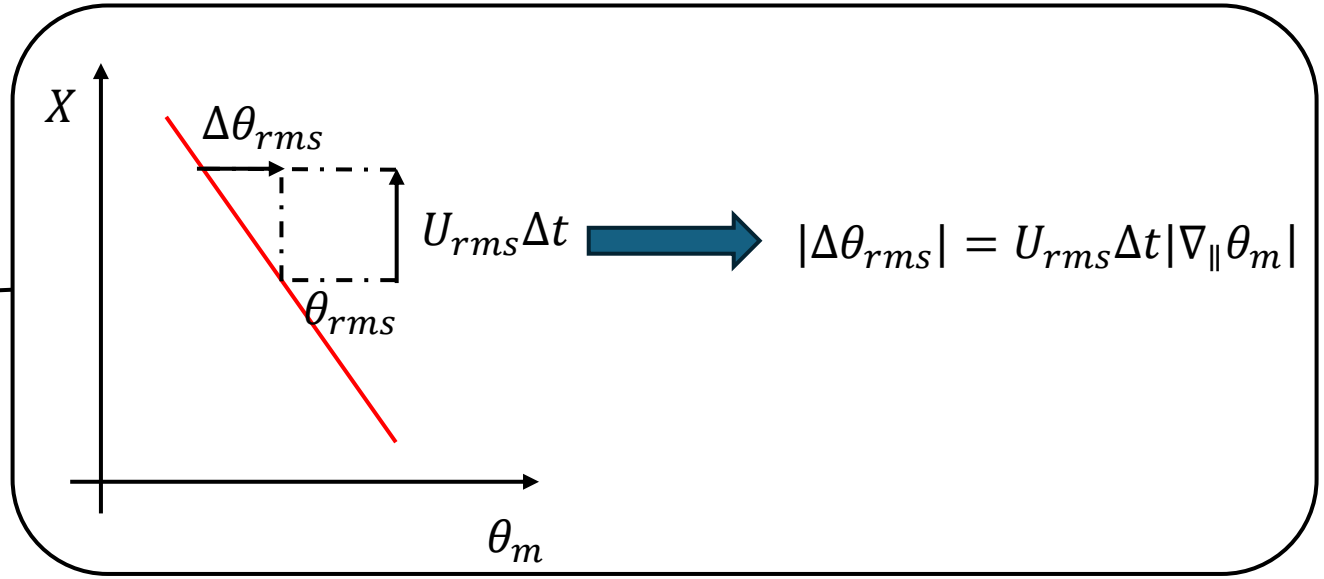
Diffusive Convection : $\tau_{ps} < \tau_{p\theta}$ → $\frac{J^h}{\tau_{p\theta}}$ ↓

Mixing length theory(fine-scale observation)

- **If the tracer variance generation is based on the local mean gradient ,**

$$\left|\frac{\partial \theta_{rms}}{\partial t}\right| \approx U_e \left|\nabla_{\parallel} \theta_m\right|$$

(U_e: Representative eddy velocity
θ_{rms}: The RSM value of the potential temperature)



- **Mixing length L_{mix}**

$$\theta_{rms} \approx U_{rms}\tau_c\left|\nabla_{\parallel}\theta_m\right| \Rightarrow L_{mix} = U_{rms}\tau_c = \frac{\theta_{rms}}{\left|\nabla_{\parallel}\theta_m\right|}$$ (τ_c :correlation time length among eddy velocity)

- **Effective lateral diffusivity K_e^{fine}**

$$K_e^{fine}\left|\nabla_{\parallel}\theta_m\right| = \overline{u'\theta'} \sim c_e U_{rms}\theta_{rms} \Rightarrow \boldsymbol{K_e^{fine} = c_e U_{rms} L_{mix}}$$

($c_e \sim 0.16$: $mixing\ effici$)

Triple decomposition framework (microscale observation)

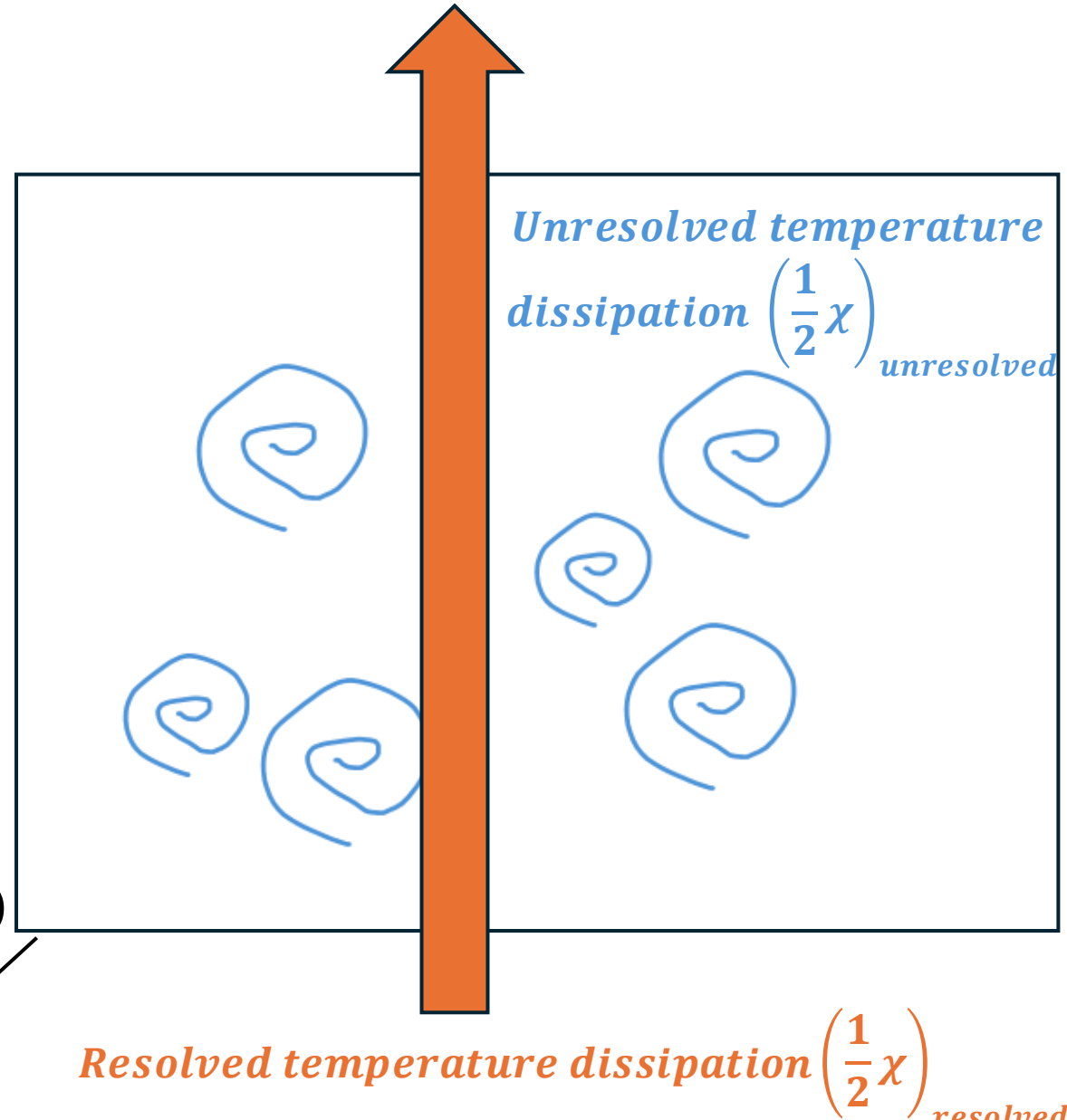
Separation of the "microscale temperature-variance dissipation rate $\left(\frac{1}{2}\chi\right)_{total}$"

$$\left(\frac{1}{2}\chi\right)_{total} = \left(\frac{1}{2}\chi\right)_{resolved} + \left(\frac{1}{2}\chi\right)_{unresolved} \quad (1)$$

Down gradient assumption

$$\begin{cases} \left(\frac{1}{2}\chi\right)_{resolved} \approx K_h\left(\frac{\partial\theta_m}{\partial z_\perp}\right)^2 \approx K_\rho\left(\frac{\partial\theta_m}{\partial z_\perp}\right)^2 & (2) \\ \left(\frac{1}{2}\chi\right)_{unresolved} \approx \left\langle K_h\left(\frac{\partial\theta_e}{\partial z_\perp}\right)^2\right\rangle \approx \left\langle K_\rho\left(\frac{\partial\theta_e}{\partial z_\perp}\right)^2\right\rangle & (3) \end{cases}$$

Under strong turbulence: $K_h \approx K_\rho$



・**Dissipation rate expressed by downgradient assumption**

$$\underbrace{\left(\frac{1}{2}\chi\right)_{total}}_{\text{Total}} = \underbrace{K_\rho\left(\frac{\partial\theta_m}{\partial z_\perp}\right)^2}_{\text{Resolved temperature}} + \underbrace{\left\langle K_\rho\left(\frac{\partial\theta_e}{\partial z_\perp}\right)^2\right\rangle}_{\text{Unresolved temperature}} \quad (4)$$

Here, we assume a 1:1 balance between the unresolved temperature dissipation rate $\left\langle K_\rho\left(\frac{\partial\theta_e}{\partial z_\perp}\right)^2\right\rangle$ and its production rate $K_e\left|\nabla_\parallel\theta_m\right|^2$.

$$\left(\frac{1}{2}\chi\right)_{total} = K_\rho\left(\frac{\partial\theta_m}{\partial z_\perp}\right)^2 + K_e\left|\nabla_\parallel\theta_m\right|^2 \quad (5)$$

Therefore, if the observed $\left(\frac{1}{2}\chi\right)_{total}$ is available , effective lateral diffusivity K_e can be estimated.
Solving for K_e , we obtain

$$K_e = \frac{\left(\frac{1}{2}\chi\right)_{total} - K_\rho\left(\frac{\partial\theta_m}{\partial z_\perp}\right)^2}{\left|\nabla_\parallel\theta_m\right|^2} \quad (6)$$

Ex) $\theta_m = 20km$ filtered out temperature
K_e = Diffusivity induced by sub $20km$ size structures